PSU Journal of Engineering, Technology and Computing Sciences
Vol. 1 No.1, pp. 8-14, December 2017

Setting – Up Equations among Parameters of Discrete Normal Random Variables

RODELIO M. GARIN
Pangasinan State University, Asingan, Pangasinan
rodgarin36@gmail.com

Abstract — **This study aims to setting – up equation among discrete normal random variables. This study was carried out by a process of investigations. The study used four stages of mathematical investigation namely: executing experiment, search for pattern, setting up equation, and testing the equation. Concerning the result, the following equations were derived from the experiment with ffaces and tossed t times: (1) the formula for the number of outcomes is** $nK = f^t$**. (2) The number of possible sum is** $= (f-1)t + 1$**. (3)The expected value of the sum is** $E(X) = \frac{t(f+1)}{2}$**. (4) Lastly, the variance of the sum is** $var(X) = \frac{\left[\frac{5}{6}f^3 - 3f^2 + \frac{25}{6}f - 2\right]t}{f^2}$**. The equations formulated were verified correct using illustrations. Therefore, the study recommends the use of the formulas, specifically in a discrete normal random variable.**

Keywords — Investigate; Discrete; Variable; Equation

I. INTRODUCTION

Probability is the measure of the likelihood that a specific event will occur. The probability of an event is mostly defined as a number between one and zero [1]. An experiment with numerical possible outcome associated with a probability is called random variable. A random variable is discrete if the numerical possible outcomes are integers, finite, and countable [2]. A normal probability distribution gives a bell-shaped curve, when plotted, symmetric about the mean, and tails in both sides extent indefinitely [3]. A normal discrete random variables are random variables whose distribution is also most like a bell – curve, symmetric about the mean (Expected value), but its tails doesn't extent indefinitely. The curve of the normal discrete random variables is a polygon, this means that it is bounded by the line segments. Tails do not extent indefinitely because the possible outcomes are finite and countable.

This study aims to determine a general formula for the number of outcome, number of possible sum, and expected value and variance of the sum of a normally distributed discrete random experiment. The number of outcomes refers to the number of possible values that may occur when the experiment executed. When an experiment executed and repeated *n* times (two or more) the sum of those values that will appear is called the possible sum. The expected value is the theoretical mean of the possible sum. The theoretical mean may happen in a long run, that is, repeating the experiment 30 or more times (central limit theorem). The variability refers to the dispersion of the possible sums from the expected value. The formula that will derive from this study can be used to determine those described parameters.

Determining the expected value and variance of a discrete random experiment is a laborious process especially to the experiments with large *n* (number of possible outcomes) and repeated *t* times. There are other processes to determine those parameters like act it out and make an organized listing then search for pattern. Each process requires more time to solve the parameters. Every time the number of possible outcomes in the experiment change, you will need another organized listing then search for pattern to determine the expected value and variance. The derive formulas were determined using listing and then search for a pattern considering the number of possible outcomes and the number of trials.

The formula can be applied in the experiment like tossing coins, rouletting wheels, rolling dice, shuffling cards, and the like. These experiments are common examples and problems in probability books. The problems differ on the number of possible outcomes and number of trials. This study provides a general formula for those experiments with *n* possible outcomes and *t* trials.

II. METHODOLOGY

Design

This study was carried out by mathematical investigation. Mathematical investigation is an approach to increase emphasis of problem solving processes. It is considered as a tool for mathematical processes in the learner and an exercise of intellect to unearth mathematical structure from a simple starting point [4].

Procedure

This study used four stages of mathematics of Investigation (see figure 1). The first stage is **executing random experiment** related to discrete normal random variable. During this stage, the random experiment will be identified then executed three or more times. The result of the experiment was listed systematically then presented using table.

Searching Pattern. In this stage the degree of difficulty of the experiment was extended by adding number of faces or categories to the random variable. On this stage, the degree of organization of the data was more sophisticated because the table consisted f rows and t trials. The data were examined critically to determine a pattern.

Setting up equation. The entries in the tables were filled up using arithmetic sequence, geometric sequence and difference method. The resulting table was examined critically to set up an equation.

Testing the equation. This stage, the formulas generated was tested in consistency by giving an additional illustration. The illustration may support the generated formula or provide a counter – example indicating the need to revise or reject the equation.

Figure 3 shows the flow chart of the investigation used in this study.

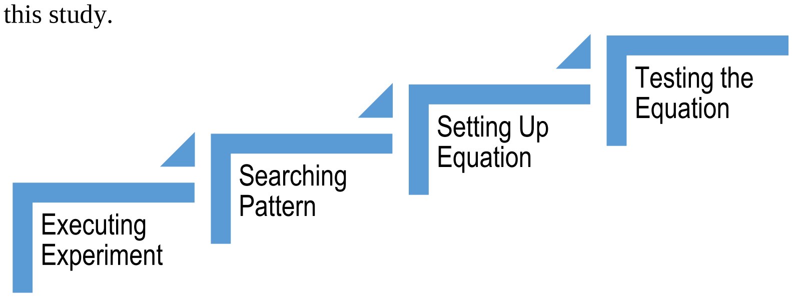


Figure 3. Flow Chart of the investigation

III. RESULTS and DISCUSSION

A roulette wheel with three sectors numbered 1, 2, and 3 spins twice, the number of possible outcome is 9. Consequently, if the roulette wheel spins thrice the number of outcomes is 27. Figure 3.1 and figure 3.2 illustrate the possible outcomes when a roulette wheel spins twice and thrice respectively.

3	1,3	2,3	3,3
2	1,2	2,2	3,2
1	1,1	2,1	3,1
	1	2	3

Figure 3.1. List of Possible Outcomes

3,3	3,3,1	3,3,2	3,3,3
3,2	3,2,1	3,2,2	3,2,3
2,3	2,3,1	2,3,2	2,3,3
3,1	3,1,1	3,1,2	3,1,3
1,3	1,3,1	1,3,2	1,3,3
2,2	2,2,1	2,2,2	2,2,3
2,1	2,1,1	2,1,2	2,1,3
1,2	1,2,1	1,2,2	1,2,3
1,1	1,1,1	1,1,2	1,1,3
	1	2	3

Figure 3.2. List of Possible Outcomes

ISSN 2599-462X(Print) *www.psurj.org/jetcs* ISSN 2599-4638 (Online)

Supposed a die with ten faces numbered 1,2,3,10 will be tossed 5 times. How many outcomes are there? Solving this problem using the procedure above is a laborious process. It needs longer time to determine correctly the answer. Table 3.1 shows the formula, to answer this question.

Table 3.1 Number of outcomes of a normal discrete random variable with f faces and t trials

		Number of trials (t)							
		1	2	3	4	5	6	⋯	t
Number of faces (f)	1	**1**	**1**	**1**	**1**	**1**	**1**	⋯	1^t
	2	**2**	**4**	**8**	**32**	**64**	**128**	⋯	2^t
	3	**3**	**9**	**27**	**81**	**243**	**729**	⋯	3^t
	4	**4**	**16**	**64**	**256**	**1024**	**4096**	⋯	4^t
	5	**5**	**25**	**125**	**625**	**3125**	**15625**	⋯	5^t
	6	**6**	**36**	**216**	**1296**	**7776**	**46656**	⋯	6^t
	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮
	f	f	f^2	f^3	f^4	f^5	f^6	⋯	f^t

Formula 3.1: The number of outcomes (nK) of a normal discrete random variable with f faces numbered 1,2,3,…, f and t trials is

$$nK = f^t$$

Where:

nK = number of outcomes

f = number of faces

t = number of trials

The possible sum of the sectors that will appear when a roulette wheel with three sectors numbered 1, 2, and 3 spins two times is one of the possible problems in this experiment. Figure 3.3 shows all the possible sums in this experiment.

3	3,1	3,2	3,3
2	2,1	2,2	2,3
1	1,1	1,2	1,3
	1	2	3

Figure 3.3. Possible sums in this experiment

Figure 3.3 contains five different colors orange, blue, red, green and black. Each color represents different sum. This implies that in the experiment there are five possible sums.

Illustration 3.1. Using formula 3.1 the number of outcomes when a dice with ten faces numbered 1, 2, 3,10 tossed five times is,

$$nK = f^t = 10^5 = 100{,}00$$

If the wheel spins three times, then the number of possible sum is 9. Figure 3.4, shows all of these possible sums.

3,3	3,3,1	3,3,2	3,3,3
3,2	3,2,1	3,2,2	3,2,3
2,3	2,3,1	2,3,2	2,3,3
3,1	3,1,1	3,1,2	3,1,3
1,3	1,3,1	1,3,2	1,3,3
2,2	2,2,1	2,2,2	2,2,3
2,1	2,1,1	2,1,2	2,1,3
1,2	1,2,1	1,2,2	1,2,3
1,1	1,1,1	1,1,2	1,1,3
	1	2	3

Figure 3.4. Possible sums in this experiment

There are seven colors in Figure 3.4 and each color represents different sums. This implies that there are seven possible sums.

Supposed a die with six faces numbered 1, 2, 3, …, 6 tossed 7 times. How many possible sums are there? This query will be answered easier if you will use formula 3.2. This formula is derived using table 3.2.

Table 3.2 Number of sums of a normal discrete random variable with f faces and t trials

		Number of trials (n)							
		1	2	3	4	5	6	⋯	t
Number of faces (f)	1	**1**	**1**	**1**	**1**	**1**	**1**	⋯	1
	2	**2**	**3**	**4**	**5**	**6**	**7**	⋯	$t+1$
	3	**3**	**5**	**7**	**9**	**11**	**13**	⋯	$2t+1$
	4	**4**	**7**	**10**	**13**	**16**	**19**	⋯	$3t+1$
	5	**5**	**9**	**13**	**17**	**21**	**25**	⋯	$4t+1$
	6	**6**	**11**	**16**	**21**	**26**	**31**	⋯	$5t+1$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮
	f	$f-0$	$2f-1$	$3f-2$	$4f-3$	$5f-4$	$6f-5$	⋯	$tf-(t-1) = (f-1)t + 1$

Illustration 3.2: The number of possible sum (nS) of a normal discrete random variable with f faces numbered 1,2,3,…, f and t trials is

$$nS = (f-1)t + 1$$

where:

S = number of possible sum

f = number of faces

t = number of trials

Illustration 3.2 The number of possible sum when a die with six faces numbered 1, 2, 3, 4, 5, 6 tossed five times is,

$$nS = (f-1)t + 1 = (6-1)5 + 1 = 26$$

Let X be the possible sum in the experiment spinning roulette wheel with three faces numbered 1, 2, and 3 three times. The variable X is an example of normal discrete random variable. Table 3.3 and figure 3.5 shows the probability distribution and the graph of the random variable X. The graph is symmetric with respect to the vertical line across the middle value in the x axis. The middle value in the x-axis is the expected sum of this experiment. This means that the graph of the Discrete Random Variable X is symmetric to the expected sum.

Table 3.3 Probability Distribution of the Discrete Random Variable X

X	3	4	5	6	7	8	9
$g(x)$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{7}{27}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{1}{27}$

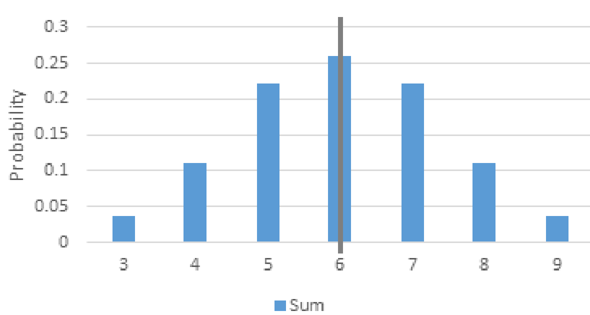
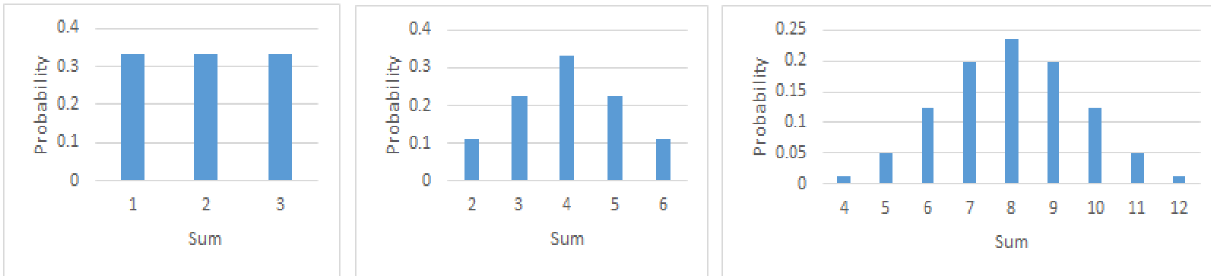


Figure 3.5. Graph of the Discrete Random Variable X.

Figure 3.6 shows the graph of an experiment spinning roulette wheel with three faces once, twice, and four times. These graphs, only figure 3.6a has a uniform distribution, otherwise has a normal distribution. The middle value of those distributions is there expected value.



a b c

Figure 3.6 Graph of roulette wheel with three faces spins once, twice, and four times.

Supposed X be the sum of the up faces when a die is rolled six times. What is the expected sum of the random variable X? This query may be solved by listing all the possible sums then find the median or mean. The data below shows all the possible sums, the median of this data is 21.

6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36

$$Median = \left[\frac{N+1}{2}\right]^{th} = \left[\frac{32}{2}\right]^{th} = 16^{th}$$

$$Median = 21$$

Therefore, $E(X) = 21$.

This query may be solved using the table 3.4 or formula 3.3. The table also shows that the expected value of the random variable X is 21.

Table 3.4 Expected Value of a Normal Discrete Random Variable with f Faces and t Trials

		Number of Trials (t)							
Number of faces (f)		1	2	3	4	5	6	…	t
	1	1	2	3	4	5	6	…	t
	2	1.5	3	4.5	6	7.5	9	…	$\frac{3t}{2}$
	3	2	4	6	8	10	12	…	$2t$
	4	2.5	5	7.5	10	12.5	15	…	$\frac{5t}{2}$
	5	3	6	9	12	15	18	…	$3t$
	6	3.5	7	10.5	14	17.5	21	…	$\frac{7t}{2}$
	F	$\frac{f+1}{2}$	$\frac{2(f+1)}{2}$	$\frac{3(f+1)}{2}$	$\frac{4(f+1)}{2}$	$\frac{5(f+1)}{2}$	$\frac{6(f+1)}{2}$		$\frac{t(f+1)}{2}$

Formula 3.3: The expected value $E(X)$ of a normal discrete random variable with f faces numbered 1,2,3,…,f and t trials is

$$E(X) = \frac{t(f+1)}{2}$$

Where:

$E(X)$ = Expected sum

f = number of faces

t = number of trials

Illustration 3.3 The expected value of the sum if a die with six faces numbered 1, 2, 3, 4, 5, 6 tossed six times is,

$$E(X) = \frac{t(f+1)}{2} = \frac{6(6+1)}{2} = \frac{6(7)}{2}$$

$$= 21$$

Illustration 3.4 The variance of a normal discrete random variable defined as "the sum of the selected sectors when the roulette wheel spins thrice" is,

X	3	4	5	6	7	8	9
X^2	9	16	25	36	49	64	81
$g(x)$	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{6}{27}$	$\frac{7}{27}$	$\frac{6}{27}$	$\frac{3}{27}$	$\frac{1}{27}$

$$E(X) = 3\left(\frac{1}{27}\right) + 4\left(\frac{3}{27}\right) + 5\left(\frac{6}{27}\right) + 6\left(\frac{7}{27}\right) + 7\left(\frac{6}{27}\right) + 8\left(\frac{3}{27}\right) + 9\left(\frac{1}{27}\right) = 6$$

$$E(X^2) = 9\left(\frac{1}{27}\right) + 16\left(\frac{3}{27}\right) + 25\left(\frac{6}{27}\right) + 36\left(\frac{7}{27}\right) + 49\left(\frac{6}{27}\right) + 64\left(\frac{3}{27}\right) + 81\left(\frac{1}{27}\right) = \frac{342}{9}$$

$$var(X) = \frac{342}{9} - 6^2 = 2$$

Using formula 3.4 is another way to solve this problem. This was derived by critically examining patterns in table 3.5.

Table 3.4 Variance of the Sum of a Normal Discrete Random Variable with f Faces and t Trials

		Number of Trials (t)							
		1	2	3	4	5	6	…	t
Number of faces (f)	1	0	0	0	0	0	0	…	$\frac{0t}{1^2}$
	2	$\frac{1}{2^2}$	$\frac{2}{2^2}$	$\frac{3}{2^2}$	$\frac{4}{2^2}$	$\frac{5}{2^2}$	$\frac{6}{2^2}$	…	$\frac{t}{2^2}$
	3	$\frac{6}{3^2}$	$\frac{12}{3^2}$	$\frac{18}{3^2}$	$\frac{24}{3^2}$	$\frac{30}{3^2}$	$\frac{36}{3^2}$	…	$\frac{6t}{3^2}$
	4	$\frac{20}{4^2}$	$\frac{40}{4^2}$	$\frac{60}{4^2}$	$\frac{80}{4^2}$	$\frac{100}{4^2}$	$\frac{120}{4^2}$	…	$\frac{20t}{4^2}$
	5	$\frac{50}{5^2}$	$\frac{100}{5^2}$	$\frac{150}{5^2}$	$\frac{200}{5^2}$	$\frac{250}{5^2}$	$\frac{300}{5^2}$	…	$\frac{50t}{5^2}$
	6	$\frac{105}{6^2}$	$\frac{210}{6^2}$	$\frac{315}{6^2}$	$\frac{420}{6^2}$	$\frac{525}{6^2}$	$\frac{630}{6^2}$	…	$\frac{105t}{6^2}$
	⋮								⋮
	f								$\frac{\left[\frac{5}{6}f^3 - 3f^2 + \frac{25}{6}f - 2\right]t}{f^2}$

Formula3.4: The variance $var(X)$of the sum of a normal discrete random variable with f faces numbered 1,2,3,…,f and t trials is

$$var(X) = \frac{\left[\frac{5}{6}f^3 - 3f^2 + \frac{25}{6}f - 2\right]t}{f^2}$$

Where

$var(X) = variance of the sum$

f = number of faces

t = number of trials

Illustration 3.5: Using formula 3.4 the variance of the variable X defined in Illustration 3.5 is,

$$var(Y) = \frac{\left[\frac{5}{6}(2^3) - 3(2^2) + \frac{25}{6}(2) - 2\right]8}{2^2} = \frac{\left[\frac{40}{6} - 12 + \frac{50}{6} - 2\right]8}{2^2} = \frac{(1)8}{2^2} = 2$$

$$var(X) = \frac{\left[\frac{5}{6}f^3 - 3f^2 + \frac{25}{6}f - 2\right]t}{f^2} = \frac{\left[\frac{5}{6}(3^3) - 3(3^2) + \frac{25}{6}(3) - 2\right]3}{3^2} = 2$$

Illustration 3.6. The variance of the variable Y defined as "number of heads occur when a coin tossed 8 times" is,

$$var(Y) = \frac{\left[\frac{5}{6}(2^3) - 3(2^2) + \frac{25}{6}(2) - 2\right]8}{2^2} = \frac{\left[\frac{40}{6} - 12 + \frac{50}{6} - 2\right]8}{2^2} = \frac{(1)8}{2^2} = 2$$

IV. CONCLUSIONS

Through the process of critical investigation, the formulas below regarding number of possible outcomes, number of possible sums, expected value and variance of the normal discrete random variable were created. These formulas were proven using illustrations.

$nK = f^t$

Number of outcomes

$nS = (f-1)t + 1$

Number of possible sums

$$E(X) = \frac{t(f+1)}{2}$$

Expected value

$$var(X) = \frac{\left[\frac{5}{6}f^3 - 3f^2 + \frac{25}{6}f - 2\right]t}{f^2}$$

Variance

Where:

f = number of faces

t = number of trials

V. RECOMMENDATION

All formulas were derived by observing patterns in the listed entries of the table consisting number of faces as rows and number of trials as columns and reliably verified using illustrations. Therefore, teachers teaching math subjects with discrete random variable distribution as a topic may use those formulas to solve problems related to this topic.

VI. REFERENCES

[1] Larson, R. & Farber, B. (2012).*Elementary Statistics: picturing the world 5th ed.* Prentice Hall.

[2] Healey, J. F. (2012). *Statistics a tool for social research 9th Ed*. Wadsworth, Cengage Learning.

[3] Mann, P.S. (2011). *Introductory Statistics 7th ed*. John Wiley & Sons, Inc.

[4] Paul Zeitz, P. (2007). The Art and Craft of Problem Solving 2nd Edition.John Wiley & Sons, Inc.

ISSN 2599-462X(Print) *www.psurj.org/jetcs* ISSN 2599-4638 (Online)